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# How Not to Enhance the Indispensability Argument†

Russell Marcus

Department of Philosophy, Hamilton College, Clinton, N.Y. 13323, U.S.A.
E-mail: rmarcus1@hamilton.edu

**ABSTRACT**

The new explanatory or enhanced indispensability argument alleges that our mathematical beliefs are justified by their indispensable appearances in scientific explanations. This argument differs from the standard indispensability argument which focuses on the uses of mathematics in scientific theories. I argue that the new argument depends for its plausibility on an equivocation between two senses of explanation. On one sense the new argument is an oblique restatement of the standard argument. On the other sense, it is vulnerable to an instrumentalist response. Either way, the explanatory indispensability argument is no improvement on the standard one.

## 1. EXPLANATIONS AND THEORIES

As a preliminary, I make two claims. The first claim is relatively uncontroversial: not all uses of mathematics should compel our belief in mathematical objects. Consider the claim: 'There are three mangoes on the table.' This claim uses a mathematical term, 'three'. A naïve argument to the existence of mathematical objects, let us call it the applicability argument, concludes that such mathematical terms should be taken to refer to numbers. It follows fairly directly from the applicability argument that there are abstract objects located outside of space and time and inaccessible to sense perception. The applicability argument, were it sound, would quickly settle core debates in epistemology over the existence of *a priori* justification as well as metaphysical debates about *abstracta.*

But the applicability argument is too weak to achieve such lofty goals. Our adjectival uses of mathematical terms are easily understood as non-referring terms. Sentences which employ them can be seen as convenient shorthand for less extravagant claims like: 'Here is a mango, and here is another mango unidentical to the prior one, and

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here is another mango unidentical to the prior mangoes.' Given the easy availability of such alternatives, we need no mathematical objects to account for uses of mathematical terms like the 'three' in the claim above.

Rewriting our casual sentences to speak austerely is a common practice. Quine [1960, p. 244] speaks insightfully about sakes and behalves in this regard. When we want to display our serious commitments, we speak soberly, invoking parsimony and rewriting our loose talk. We reflectively eliminate references to point-masses and frictionless planes as well as adjectival uses of natural-number terms.

The weak applicability argument should thus be distinguished from the stronger indispensability argument. The latter argument, which I specify carefully below, is that the applications of mathematics in science justify beliefs in mathematical objects because acceptable reformulations of important claims which involve mathematics are unavailable. In order to justify mathematical beliefs, indispensabilists ordinarily appeal to uses of mathematics which are more difficult to rephrase than adjectival uses of small natural numbers. They invoke uses of real numbers for measurement, for example, or the employment of functions, curved space-time geometries, or probability distributions. The indispensability argument depends for its plausibility not merely on the claim that we use mathematics, but on the claim that we cannot in any attractive way rewrite our theories to eliminate those uses.

I make no claim here about whether there are mathematical terms used in scientific theories (or anywhere else) which can not be attractively eliminated. I also do not claim that so-called nominalistic reformulations of scientific theories have the importance that they are sometimes purported to have: that they undermine our beliefs in mathematical objects. My first claim is just that some statements which use mathematical terms are naturally and reasonably seen as convenient shorthand for more complicated but less extravagant statements which do not contain such references. Some ordinary uses of mathematical terms are so clearly eliminable and so minimally mathematical that any argument for the existence of mathematical objects which invokes those uses is pointlessly weak.

My second preliminary claim may be less well-traveled: we can distinguish two incompatible senses of explanation which I will call metaphysical and epistemic. Metaphysical explanations are ones in which we speak most seriously in order to express the deep structure of the world. The Deductive-Nomological (D-N) model of explanation is mainly metaphysical, as are many more recent and more plausible alternatives like the unificationist model or the causal-mechanical model. We evaluate such models largely on how well they represent the way the world works.

Epistemic explanations, in contrast, aim at increasing the understanding of an individual. Such explanations, or aspects of explanation, may include claims which are useful for that purpose without being true.[1] The philosophical literature on explanation is



[1] Brown [2012] distinguishes these two senses of 'explanation' without naming them. He argues to a similar end of undermining the indispensability argument, but along a different route. Salmon [1984] distinguishes epistemic, modal, and ontic explanations and traces that distinction to Aristotle. Salmon focuses on the D-N model to characterize epistemic explanation because of the expectations raised in us by considering D-N inferences; I take D-N explanations to be metaphysical because of

messy in part because of a natural tension between metaphysical and epistemic senses; I will take a moment to show how.

Explanations are ordinarily taken to be claims or inferences. But whether a claim or an inference is an explanation seems, at times, contextual or pragmatic. What one person takes as explanatory may be incomprehensible, thus not explanatory, for another. For example, while one can say that the principles of general relativity explain gravitation, they do no such thing for most of us who do not understand the fundamental laws. Still, on plausible models of explanation, general relativity may well explain gravitation.

On the paradigmatically metaphysical D-N, or covering-law, account, the explanation of a state of affairs is an inference involving the laws of a serious theory combined with appropriate initial conditions. We speak most strictly in such theories, a criterion which Hempel and Oppenheim [1948, p. 248] call the empirical condition of adequacy. Railton's [1978] model of probabilistic explanation and Kitcher's unificationist account work similarly. [Kitcher, 1991], for example, invokes unifying argument patterns which also answer why questions with inferences made by a serious theory. Salmon's causal-mechanical model focuses explanations on real causal processes as opposed to mere statistical generalities.

For proponents of such metaphysical models, the general principles and particular claims invoked by an explanation should be true (or empirically correct). Ordinary explanations may be either shorthand for proper explanations or loose invocations of the term. Proponents of metaphysical models of explanation aim to increase understanding for an ideal or sufficiently educated reasoner. But the central focus of such theories is on how they represent the world, not on how they foster people's understanding. Railton, for example, promises 'An account of probabilistic explanation *free from relativization to our present epistemic situation*' [Railton, 1978, p. 219, emphasis added].

Indeed, proponents of metaphysical theories of scientific explanation sometimes denigrate that aspect of explanation which concerns actual human cognition or understanding as irredeemably psychological. (See [Friedman, 1974, p. 7].) The proponent of a metaphysical theory may promise that when we understand the laws or causal structures or unifying principles underlying a phenomenon, we will understand why it occurred. But the desire to provide an objective account of explanation independent of any particular agent remains strong and attempts to account for human understanding within such a model fall short.

In contrast, ordinary explanations may increase our understanding while appealing to casual, unserious claims. When I explain the presence of three mangoes on the table as the result of my bringing four but your having eaten one, or when I explain my actions as having been done for someone else's sake, I may successfully communicate using language which does not reflect the ultimate structure of the world. Explanations in this epistemic sense are often agnostic concerning their commitments, including mathematical ones. They can invoke mathematical terms which may be interpreted

their employment of empirically adequate laws. Another difference: Salmon focuses on causation for ontic explanation, a focus which precipitously rules out mathematical explanations.

variously by platonists, fictionalists, or those who believe that mathematical terms are oblique references to other things, like possible arrangements of concrete objects.

Explanations may be casual in other ways. They may refer to idealizations.[2] They may explain the height of a flagpole by the length of its shadow. They may invoke models which work, like any metaphor, only so far. We do not think that the atom is literally constructed like the solar system; nevertheless, the image of electron orbits can be a useful heuristic. Such explanations may be perfectly good answers to ordinary why questions. They need not track the fine structure of the universe.

So we have two distinct senses of 'explanation'. Epistemic explanations, and epistemic aspects of explanation, may be independent of the way the world is without failing, for that reason, to be explanatory. Criteria for good epistemic explanations include intelligibility to a particular audience and familiarity. They vary with the audience. We need not take all references in an epistemic explanation seriously, though they may represent the world accurately in some ways.[3]

In contrast, criteria for good metaphysical explanations include getting at the right laws and general principles of the world. They transcend their audience. They are apt for expressing what we think exists. We must take the references of their terms seriously.

It is quixotic to try to capture the contrasting kinds of explanation with a single account. The essential tension in our concept forces any account toward either actual human understanding or expressing the nature of the world. Moreover, an explanation in which we are most serious about our references may not be useful when we want to explain, in the epistemic sense, facts about the world. The degree to which a theory is explanatory, whether or not it includes mathematics, may not be proportional to the degree to which we should believe in the objects to which it refers.

The thesis of this paper is that the new explanatory indispensability argument is no improvement on the standard argument because it depends for its plausibility on an equivocation between the metaphysical and epistemic senses of 'explanation'. In order to make that case, let us look at both the standard and explanatory versions of the argument.

## 2. THE STANDARD INDISPENSABILITY ARGUMENT, DISPENSABILISTS, AND WEASELS

There is no standard, canonical version of the indispensability argument.[4]

We can use Quine Indispensability Argument (QIA):

QIA1 We should believe the theory which best accounts for our sense experience.



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[2] Batterman [2003] argues convincingly that much of scientific discourse consists of what he calls asymptotic reasoning, using highly idealized models.

[3] An utterly fictitious account of a phenomenon would be useless. An explanation must hook on to the world in some way. But the degree to which an epistemic explanation must represent the world accurately is an interesting and open question beyond the scope of this paper.

[4] The argument is often called the Quine-Putnam indispensability argument. See Quine [1939; 1948; 1951; 1955; 1958; 1960; 1978; 1986] and Putnam [1962; 1971; 1975; 1994].

QIA2 If we believe a theory, we must believe in its ontological commitments.
QIA3 The ontological commitments of any theory are the objects over which that theory first-order quantifies.
QIA4 The theory which best accounts for our sense experience first-order quantifies over mathematical objects.
QIAC We should believe that mathematical objects exist.

Any indispensability argument for mathematical beliefs is vulnerable to attacks on the premise which, like QIA4, alleges that mathematics is ineliminable from science. Field [1980] shows how to reformulate Newtonian gravitational theory (NGT), trading mathematical axioms for claims about the physical structure of space-time. John Burgess [1984; 1991a; 1991b] refined and improved on Field's work. Charles Chihara [1990] and Geoffrey Hellman [1989] rewrote mathematical theories as modal ones. Mark Balaguer attempted to show how quantum mechanics could be nominalized. We can call these projects, generally, dispensabilist. The current consensus about dispensabilist projects is that something close to Field's project can work for NGT, but other theories, including those based on curved space-time and those which rely on statistical frameworks, are resistant. No neat first-order theory which eschews mathematical axioms is likely to suffice for all of current and future science. But the lack of dispensabilist strategies currently available is weak evidence for their impossibility and the dispensabilist has reasonable hope of finding moderately attractive reformulations of large swaths of scientific theory.[5]

Mathematical instrumentalists provide a different attack on the indispensability argument. They argue that our uses of mathematics, even in our best theories, should not be taken seriously. Instrumentalists accept that mathematics is essential to science but deny that applications of mathematics are relevant to the justification of our mathematical beliefs. In particular, Joseph Melia claims that one can 'weasel out' of the indispensability argument, urging we can interpret both mathematicians and scientists as taking back all *prima facie* commitments to *abstracta* whether or not they can be attractively eliminated.[6] Melia defends weaseling by claiming that while scientists use mathematics in order to express facts which are not representable without mathematics, such representations are not serious.

Versions of the indispensability argument vary in their liability to weaseling. QIA resists weaseling with Quine's insistence on specifying how and when we are to be taken as speaking seriously. These details arise out of a combination of his holism and his naturalism, as well as his methods for representing our commitments. Quine's argument not only insists that evidence transfers from science to mathematics and that our scientific theories are the locus of our commitments, but also that we find our commitments in a particular way, one which is in principle acceptable to both nominalists and platonists. When the weasel says that we can differentiate between the real and the merely instrumental posits of our theory, Quine's holism blocks the move: all posits are on a par. When the weasel says that we can take back portions of what we say, Quine's naturalism denies that such double talk is defensible.



[5] See [MacBride, 1999] and [Burgess and Rosen, 1997], especially p. 118.

[6] See [Melia, 2000, p. 457; 2010, p. 1119]. Other versions of the weaseling strategy include [Yablo, 2005; Leng, 2002; 2005; 2010; Pincock, 2004a; 2004b]; and [Balaguer, 1998, Ch. 5].

Quine's arguments against double talk appear throughout his work and are essential to the strength of QIA. For Quine, if our best theory requires electrons for its bound variables, then we should believe in electrons. If it requires sets, we are committed to sets. Quine's response to Carnap's internal/external distinction relies on the illegitimacy of double talk: once one has accepted some language as an internal matter, one can not dismiss its commitments as merely conventional. Quine's response to the Meinongian Wyman in 'On what there is' [1948], is similar: he distinguishes between the meaningfulness of 'Pegasus' and its reference in order to avoid admitting that Pegasus subsists while denying that Pegasus exists. Putnam, defending Quine's indispensability argument, makes the double-talk argument explicitly. 'It is silly to agree that a reason for believing that *p* warrants accepting *p* in all scientific circumstances, and then to add "but even so it is not *good enough*" ' [Putnam, 1971, p. 356].

Worries about double talk bother Quine's critics, too: 'If one *just* advocates fictionalism about a portion of mathematics, without showing how that part of mathematics is dispensable in applications, then one is engaging in intellectual doublethink . . . ' [Field, 1980, p. 2].

Weaseling is taking back what one initially alleges. So an indispensability argument which explicitly and reasonably rejects the legitimacy of double talk can resist weaseling. Of course, no arguments will convince a mulish nominalist to accept the existence of mathematical objects. The instrumentalist can always baldly deny that mathematical beliefs are justified. But the standard argument at least provides a defense. QIA says that our commitments are to be found exclusively and uniformly in the quantifications of our best theory. Once we differentiate among the posits of a theory, between the real and the merely instrumental, we are rejecting the cornerstones of QIA which block the weasel.

## 3. THE EXPLANATORY ARGUMENT

According to the new explanatory indispensability argument (EIA), we should believe in mathematical objects because of their indispensability, not to scientific theories, but to scientific explanations. Versions of the argument appear in recent work by Mark Colyvan [2008], Alan Baker [2005; 2009], and Sorin Bangu [2013]. I shall follow Paolo Mancosu's formulation.

EIA1 There are genuinely mathematical explanations of empirical phenomena.
EIA2 We ought to be committed to the theoretical posits postulated by such explanations.
EIAC We ought to be committed to the entities postulated by the mathematics in question [Mancosu, 2011, §3.2].[7]

Proponents of EIA are motivated both by dispensabilist criticisms of the standard argument and by concerns about what some proponents of weaseling see as the merely representational role of mathematics in science. The proponent of EIA sets

[7] Baker [2001, p. 613] calls the argument enhanced.

aside the question of whether scientific theories can be rewritten without mathematics and argues that non-mathematical explanations of physical phenomena are either unavailable or less preferable, in at least some cases.

The literature on the explanatory argument is divided between two ways to view EIA in relation to versions of the argument, like QIA, that appeal to theories. Some, like Bangu and Melia, see EIA as an additional demand on the platonist, and thus an additional option for the nominalist. They argue that even if dispensabilist constructions like those of Field are not available we should withhold commitments to mathematical objects because there are no genuinely mathematical explanations. On the Bangu/Melia view, the platonist should show mathematics to be indispensable from both theories and explanations; the nominalist need only show that mathematics is eliminable from explanations or theories.[8]

Others see the argument as an additional option for the platonist, thus an additional demand on the dispensabilist. Baker and Lyon and Colyvan argue that even if dispensabilist constructions of scientific theories are available, we should believe in mathematical objects as long as there are genuinely mathematical explanations of physical phenomena. Exploring David Malament's claim that phase-space theories resist dispensabilist constructions, Lyon and Colyvan write, 'Even if nominalisation via [a dispensabilist construction] is possible, the resulting theory is likely to be less explanatory; there is explanatory power in phase-space formulations of theories, and this explanatory power does not seem recoverable in alternative formulations' [2008, p. 242]. On this view, the platonist must show that mathematics is indispensable only from explanations or theories; the nominalist must eliminate mathematics from both theories and explanations. Since, I will argue, the explanatory argument is at best an elliptical allusion to the standard argument, it will not matter here whether EIA is taken as an additional burden on the nominalist or as more work for the platonist.

## 4. MATHEMATICAL EXPLANATIONS IN SCIENCE

Debate over EIA has mainly focused on EIA1. Support for QIA, taking explanatory merit as a theoretical virtue, carries over to that premise. For example, Mark Colyvan presents three cases to show that standard mathematized theories (ME) have greater explanatory merit than their nominalist counterparts.

ME1 Bending of light. The best explanation of light bending around large objects is geometric, rather than causal.

ME2 Antipodes. The Borsuk-Ulam topological theorem, along with appropriate bridge principles, explains the existence of two antipodes in the Earth's atmosphere with the same pressure and temperature at the same time.

ME3 The Fitzgerald-Lorentz contraction. Minkowski's geometrical explanation of the contraction of a body in motion, relative to an inertial

[8] See [Melia, 1998, p. 70; Bangu, 2008, p. 14; Bangu, 2013, p. 4] and [Melia, 2002; Leng, 2005, p. 179], though working with explanation as a theoretical virtue can also be seen as taking this route.

reference frame, relies on equations in four dimensions, representing the space-time manifold.[9]

Not all of these examples are equally compellingly described as mathematical explanations of physical phenomena. Baker rightly worries about the status of the geometry on which ME1 and ME3 rely. If the relevant geometry is physical geometry, then the explanations proceed without appeal to pure mathematics and so can not justify beliefs in pure mathematical objects. Baker also argues that ME2 is a prediction rather than an explanation. There is no antecedent why question concerning the atmospheric antipodes since we are unlikely to discover them: there are insurmountable limitations on the precision of our instruments and no independent interest in the phenomenon. Mary Leng complains that ME2 also requires contentious idealizations; the requisite principles mapping the mathematical theorem to the world will not apply.[10] Concerned about such examples, Baker produced an influential cicada case.

ME4 Cicadas. That prime-numbered life-cycles minimize the intersection of cicada life-cycles with those of both predators and other species of cicadas explains why three species of cicadas of the genus *Magicicada* share a life cycle of either thirteen or seventeen years, depending on the environment.

The phenomenon of cicadas having prime-numbered life-cycles intrigued biologists, who sought an explanation. A mathematical explanans (that prime periods minimize intersection) supports the explanandum that organisms with periodic life-cycles are likely to evolve periods that are prime. The latter is a mixed biological/mathematical law which can be used to explain the empirical claim that cicadas in a particular ecosystem-type are likely to evolve 17-year periods. Baker claims that the relevant mathematics is both indispensable and not merely representational; the number theorem plays an explanatory, rather than indexing, role [Baker, 2009, p. 614].

Bangu explores a worry about ME4, one which would hold for other examples like Colyvan's ME3. He presents four *desiderata* of examples used to support EIA1. In addition to, first, their indispensable uses of mathematics and, second, their being genuinely mathematical explanations, they should, third, be fairly simple. Since rewriting theories to avoid quantification over mathematical objects is mainly a philosopher's project, not of compelling interest to many scientists or mathematicians, relevant techniques for eliminating mathematics may not yet be developed. The indispensabilist should avoid resting the case on a lack of nominalist strategies which is due only to the difficulty of the task.

Most relevantly to the case of ME4, proponents of EIA should not, fourth, beg the question by presenting examples in which the explanandum contains ineliminable uses of mathematics. Bangu argues that the mathematical explanation in Baker's cicada

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[9] See [Colyvan, 2001, pp. 81–86]. [Colyvan, 2007, pp. 120–121] presents three further illuminating examples.

[10] See [Baker, 2005, pp. 226–227] and [Leng, 2005, pp. 181–182].

example only concerns the mathematical phenomenon, the result about prime periods, and so is question-begging as an argument for platonism.

So Bangu presents the banana game, which we can call ME5.[11] Two players compete to collect bananas by choosing among crates filled with them. By adjusting probabilities, the game can be constructed so as to ensure the victory of one side over the other. The explanandum in such cases, that one side consistently wins, has no ineliminable uses of mathematics: it is just about winning bananas. But the explananda inevitably include mathematics in the forms of probabilities and expected values.

Recent work on EIA has seen a profusion of further examples supporting EIA1, though these need not concern us here.[12] The central claim of this paper is that EIA fails because of problems in its second premise. Whether their supporting examples work exactly as proponents of EIA require is too strong a demand for establishing EIA1. The underlying plausible claim is that there are mathematical explanations of physical phenomena. It may be possible to re-describe some explanations either to eliminate or to isolate the mathematical elements, or to show that the mathematics plays a merely representational role. But as they stand, such examples provide unsurprising evidence for EIA1.

## 5. EPISTEMIC EXPLANATION AND THE EXPLANATORY INDISPENSABILITY ARGUMENT

Granting EIA1, the success of EIA depends on the claim that we should believe in the objects to which our scientific explanations refer. The difference between epistemic and metaphysical senses of 'explanation' discussed in §1 now applies. If EIA invokes a metaphysical sense of 'explanation', it does not differ from the Quinean argument. The central question is whether mathematical objects can be eliminated from our best theories: can Baker's cicada case, or other supporting examples, be written in a canonical language apt for expressing our serious commitments without mathematical terms? Projects like that of Rizza [2011], which shows how to nominalize Baker's ME4, would be apt responses. Thus, if EIA is going to be an enhanced or extended version of the indispensability argument, its proponents must appeal to a different model of explanation. But as soon as the proponent of the explanatory argument gives up the strictly metaphysical sense of 'explanation', she undermines the essential premise, EIA2, that we should believe that the terms used in our explanations refer.

To see in more detail the fruitlessness of appeal to EIA, recall the claim, from Lyon and Colyvan, that the explanatory power of phase-space formulations of theories is unrecoverable in nominalist reformulations. On any metaphysical model of scientific explanation, Lyon's and Colyvan's claim is false, *ex hypothesi*. Conserving explanatory power is a standard minimal requirement on nominalist reformulations and it works unlike other theoretical virtues. One might wonder whether sacrifices in simplicity of ideology are worth parsimony in ontology. But dispensabilists may not give up explanatory power, in the metaphysical sense, in their reformulations. Field constructs representation theorems precisely to support the claim that his reformulation

[11] For details, see [Bangu, 2012, pp. 162–173].

[12] See, for example, [Lyon and Colyvan, 2008; Mancosu, 2011; Saatsi, 2011, p. 145.].

lacks no explanatory power of the standard theory. One could nominalize a scientific theory by producing an alternative with less explanatory power only if one were using a non-metaphysical sense of the term.

One might wonder whether the proponent of EIA could trade on another equivocation on 'explanation' here. Perhaps Lyon and Colyvan (or other defenders of EIA) could argue that a dispensabilist reformulation of a standard theory need only conserve the deductive strength of the original theory and not the full explanatory merit of that theory. The reformulation could conserve, say, the Deductive-Nomological explanatory power of the original theory but not, say, the Causal-Mechanical explanatory power, or the Unificationist explanatory power, of the original. The proponent of EIA could then claim that we are justified in believing in mathematical objects because they appear in our best explanations (whatever those turn out to be) even though nominalistic reformulations of our best theories are available.[13]

I am skeptical of this proposal for two reasons. First, there are many different ways to write our theories, even in formal languages. They may be construed as first-order or second-order or something in-between, adopting, say, the mereological axioms Field uses. Theories which invoke mathematics may use a variety of different set theories or may eschew set theory in favor of direct axiomatizations of real analysis. Scientific theories themselves can be formulated variously, too; there are myriad distinct formulations of theories of electromagnetism, for example, in terms of vectors, tensors, even quaternions [Baker, 2001, p. 97]. To consider a theory *N* to be a nominalistic reformulation of some standard theory *S*, we must believe that *N* is an appropriate way to express our most sincere beliefs about the world and its constituents. But if we believe that our mathematical beliefs are justified by their appearances in an explanation *E* which is not utterly captured by *N*, then neither *N* nor *S* is the appropriate way to express our beliefs; *E* is. There is no sense in which a theory should be called a nominalistic reformulation if it fails to capture the content of the theories which compel our belief.

Second, it is difficult to see how such an *N* could conserve the deductive strength of a theory without conserving explanatory power, as Lyon and Colyvan claim. The D-N model of scientific explanation focuses on inferences from laws and initial conditions for explanations of particular events. But the D-N model is too liberal for well-known reasons such as allowing the height of a flagpole to be explained in terms of the length of its shadow. More recent and more plausible models tend to restrict the inferences classified as explanations. Thus, if a reformulation conserves deductive strength, any sub-class of inferences which are deemed explanations will be nominalized as well.

Proponents of EIA should specify which models of explanation are relevant to their claims. Whatever models they choose, though, will have to be what I call metaphysical in order to generate the claim that we should believe in the mathematical objects to

[13] This proposal was suggested by comments from a referee for this paper, though proponents of EIA are mainly silent on which concept of scientific explanation they invoke. Theories of scientific explanation do not apply in obvious ways to cases of mathematical explanation in science. And some prominent theories of explanation, like the causal-mechanical model, seem question-begging against mathematical explanation since mathematical objects are essentially non-causal. Recent work on mathematical explanation may be changing the picture; see, for example, [Baker, 2012].

which an explanation refers. On such metaphysical models, Lyon's and Colyvan's claim remains implausible.

Lyon's and Colyvan's claim is plausible though if we interpret 'explanatory power' in an epistemic sense. Unlike standard scientific theories, dispensabilist reformulations will be imperspicuous, useless to working scientists. Field grants that standard theories are more epistemically explanatory by arguing that mathematics is conservative over standard scientific theory, that adding mathematical axioms to nominalist theories will not allow one to derive any further nominalist conclusions. We will inevitably use the greater epistemic explanatory force of standard theories. Similar remarks hold for Baker's cicada case and the others. We would not consider an explanation to be a successful dispensabilist reformulation unless it retained the deductive and mathematical expressiveness of the original (*e.g.*, the concept of primeness in ME4). If we decide that a reformulation is inadequate, we must be invoking an alternative sense of 'explanation'.

We can also see that EIA depends on a non-metaphysical sense of explanation by noting that standard metaphysical accounts of scientific explanation do not apply comfortably to mathematical explanations within mathematics. Many mathematical inferences are not explanatory. We can derive '$7 + 5 = 12$' from basic axioms, but such derivations are not ordinarily taken as explanations. The amusement with which we reflect on the fact that it takes several hundred pages in *Principia Mathematica* to arrive at the proof of '$1 + 1 = 2$' speaks directly to the ways in which we take such derivations to be non-explanatory. Any evaluation of proofs as explanatory or not must take factors other than mere derivability from axioms into account. They may be explicated psychologically or by appeal to unifying proofs, for example. Mathematical explanation, whatever it might be, is not strictly about deducibility.[14]

In response to this last argument, the defender of EIA could claim that mathematical explanations of physical phenomena have two parts: a strictly mathematical explanation of a strictly mathematical theorem and a broader explanation of the physical phenomenon which invokes the mathematical theorem. Scientific explanations could remain metaphysical whatever the nature of mathematical explanation, even if there is no such coherent concept. The nature of the pure mathematical explanation could thus be isolated from, and irrelevant to, the nature of the broader explanation.

But unless broader explanations are to take mathematical results as brute facts, the nature of purely mathematical explanations is not isolable. Uses of mathematics in science naturally raise questions about why these results hold: their scope and limits and their relations to other mathematical theorems. While scientists often just want the proper formula or relevant set of differential equations, understanding the relations between one mathematical formula and another is central to understanding how and why the mathematics applies. Narrow mathematical theorems often generalize, from claims about, say, squares to claims about all polygons. The more general a theorem, the broader its applications. It is at least odd to say, as the proponent of EIA here imagined does, that we should believe in the existence of mathematical objects because

[14] [Steiner, 1978; Hafner and Mancosu, 2005; Lange, 2010] contain useful discussions of pure mathematical explanation.

they play an ineliminable explanatory role in science while dismissing the nature of mathematical explanation as irrelevant.[15]

Whether or not the nature of pure-mathematical explanation is relevant, the proponent of EIA must defend the claim that we should take our mathematical explanations of physical phenomena seriously. If they are metaphysical explanations, the proponent of EIA can adopt Quine's argument for the claim. But then EIA is no improvement on QIA. If they are epistemic explanations, the argument will be difficult to make: there is no good reason to take the references of epistemic explanations, especially the contentious mathematical references, literally. We need not be fully serious when we provide an epistemic explanation because explanations which facilitate our subjective understanding may not reveal our commitments.

If we are tempted to believe in mathematical objects by an explanation which uses mathematics, the explanation is thus not doing the work. The work is done by the background claim that there is a good theory which supports the explanation, which requires mathematics, and which we should believe. Defenders of EIA rely on a metaphysical notion of explanation to motivate the seriousness of our speech but switch to an epistemic notion of explanation to claim that there are mathematical explanations of physical phenomena. If we want our mathematical explanations to be taken literally, we should find a way to fit them into a metaphysical model so that QIA applies and the relevant dispensabilist reformulations may be considered. Alternatively, the indispensabilist can find a way to argue not merely that there are mathematical explanations of physical phenomena, like ME1–ME5, but also that such explanations are meant literally. Unless such a defense is developed, the explanatory indispensability argument is no improvement on the standard one.

## 6. WEASELING AWAY THE EXPLANATORY ARGUMENT (BUT NOT THE STANDARD ARGUMENT)

I have been arguing that a weaseling response to EIA is legitimate, on the epistemic sense of 'explanation'. Leng also argues that EIA is susceptible to weaseling, but she derives her claim from her broader arguments against QIA. Like Melia, Leng claims that mathematics merely provides a language for representing physical facts and that such representations need not be taken seriously. 'Nothing is lost in the explanation of cicada behavior if we drop the assumption that natural numbers exist' [2005, p. 186]. Leng makes similar claims against Colyvan's examples.

While weaseling claims are appropriate responses to EIA, they do not, as I argued in §2, extend to QIA. We need not speak most literally in our epistemic explanations. But it is reasonable to expect our commitments to be represented by our best and most

[15] Mancosu agrees: 'It … appears that a proper account of explanations in science requires an analysis of mathematical explanations in pure mathematics.' [2011, §3.2] Baker dissents: '[I]t seems to be quite rational not to care about whether the proofs of the mathematical results utilized by a given SDME [Science-Driven Mathematical Explanation] are explanatory. Intuitively, all that matters for the purposes of constructing an adequate SDME is that the mathematical results being used have been proved, because all the scientists are relying on is that these results are demonstrably true.' [2012, p. 263]

sincere attempts to get at the structure of the world.[16] For the proponent of QIA and its most prominent dispensabilist critics, our most sincere representation of the world is our best theory.

So my claim that we can weasel away our commitments to EIA is not the instrumentalist's stubborn denial that we should believe any reference to mathematical objects. It is the more measured claim that only in our best representations of the world can we be confident in those references. If our most serious statements include references to mathematical objects, we can not, on pain of contradiction, sincerely deny their existence. The nominalist thus may not use the aptness of weaseling to EIA as a general strategy for resisting the indispensability argument.

## 7. WHAT DO MATHEMATICAL EXPLANATIONS SHOW?

What shall we make of the fact that, in a wide range of cases, scientific explanations are more elegant and compelling if they include references to mathematical objects? The indispensabilist devises compelling examples but the weasel is really a mule, refusing to admit that any uses of mathematics are worth taking seriously. The weasel thus proposes to return to the period before Quine's work when philosophers formulated ontologies independent of the needs of scientific theory, when they questioned the existence of electrons, for example, because we could not see them directly.

The strength of Quine's indispensability argument arises from his proper insistence that we cannot invoke the physicist's theoretical commitments to electrons as reasons to believe in electrons without also being serious about the references to mathematical objects used in those theories. One's natural suspicion of the existence of abstract objects can only go so far. Melia claims that our expressive resources may be too impoverished to say what we want to say without invoking mathematics. But we must, at some point, speak seriously.

In his 1946 Harvard lecture on nominalism, Quine coins the term 'struthionism' to apply to those like Carnap (and now Melia and Leng) who refuse to take references to mathematical objects within our scientific theories literally. 'Struthionism' has the Greek word for ostrich at its core; so there is a third creature in the nominalist's menagerie: weasels, mules, and ostriches. I have defended their claims that our epistemic explanations of physical phenomena need not impel us to believe in the referents of their mathematical terms. But weaseling remains awkward despite the instrumentalist's assurances.

There is a middle ground. If we had a justification of mathematical beliefs which did not depend on the applications of mathematics in science then we could avoid seeing explanations such as ME1–ME5 as appealing to fictional objects without also taking their invocations of mathematics as grounds for our beliefs. Such a justification might invoke a platonist's intuition or a re-interpreter's ability to grasp consistency. We would believe (*ex hypothesi* truly and justifiably) the axioms of ZFC, say. We could thus use

[16] Brown [2012], denying the soundness of QIA, argues that mathematics can explain in what I call the epistemic sense, but not in the metaphysical sense. While there are good reasons to question, with Brown, even this strongest version of the indispensability argument, I leave this question open here.

mathematics in science without concern about double talk. But we would not infer our mathematical knowledge from these uses. So even if the indispensability argument gives us no reason to believe pure mathematical claims, our best explanations may still use mathematical claims sincerely.

Neither the indispensabilist nor the weasel will find such a position attractive.[17] Most weasels are motivated by commitments to nominalism. The indispensabilist wants to convince the nominalist that mathematical beliefs are justified strictly by our ordinary uses of science. Still, independent considerations for or against the existence of mathematical objects are beside the point here. I am merely noting that the weasel's denial of the inference to mathematical objects from their indispensable uses in science is compatible with the indispensabilist's claim that our best scientific explanations are more convincing if they refer only to objects we believe to be real.

I have argued that the explanatory indispensability argument trades on an equivocation in our concept of 'explanation'. If it uses an epistemic sense, its claim to justify mathematical beliefs is implausible. If it uses a metaphysical sense, then it provides the platonist with no further ammunition than the traditional Quinean argument, QIA. I did not defend QIA. In fact, I believe that problems with Quine's holism and the limitations on the platonism that the argument yields are serious. But QIA resists weaseling in ways that EIA does not. The explanatory argument thus appears to be no improvement on the standard one. Still, it is possible that there is a third way to understand mathematical explanation in science, between my metaphysical and epistemic senses, on which EIA might be more successful.

## REFERENCES


Baker, Alan [2001]: 'Mathematics, indispensability and scientific practice', *Erkenntnis* **55**, 85–116.

——— [2005]: 'Are there genuine mathematical explanations of physical phenomena?' *Mind* **114**, 223–238.

——— [2009]: 'Mathematical explanation in science', *The British Journal for the Philosophy of Science* **60**, 611–633.

——— [2012]: 'Science-driven mathematical explanation', *Mind* **121**, 243–267.

Balaguer, Mark [1998]: *Platonism and Anti-Platonism in Mathematics*. Oxford: Oxford University Press.

Bangu, Sorin Ioan [2008]: 'Inference to the best explanation and mathematical realism', *Synthese* **160**, 13–20.

——— [2012]: *The Applicability of Mathematics in Science: Indispensability and Ontology*. London: Palgrave Macmillan.

——— [2013]: 'Indispensability and explanation', *The British Journal for the Philosophy of Science*. **64**, 255–277.

Batterman, Robert [2003]: *The Devil in the Details*. Oxford: Oxford University Press.

Brown, James Robert [2012]: *Platonism, Naturalism, and Mathematical Knowledge*. London: Routledge.

Burgess, John [1984]: 'Synthetic Mechanics', *Journal of Philosophical Logic* **13**, 379–395.

——— [1991a]: 'Synthetic Physics and Nominalist Realism', in C. Wade Savage and Philip Ehrlich, eds, *Philosophical and Foundational Issues in Measurement Theory*. Hillsdale: Lawrence Erlblum Associates, 1992.


[17] Thanks to Alan Baker for stressing this point.

——— [1991b]: 'Synthetic Mechanics Revisited', *Journal of Philosophical Logic* **20**, 121–130.

Burgess, John, and Gideon Rosen [1997]: *A Subject with No Object*. Oxford: Oxford University Press.

Carnap, Rudolph [1950]: 'Empiricism, semantics, and ontology', in his *Meaning and Necessity: A Study in Semantics and Modal Logic*. 2nd ed. Chicago: The University of Chicago Press, 1956; Midway reprint, pp. 205–221. New York: Barnes and Noble, 1988.

Chihara, Charles [1990]: *Constructabiliy and Mathematical Existence.* Oxford: Oxford University Press.

Colyvan, Mark [2001]: *The Indispensability of Mathematics*. Oxford: Oxford University Press.

——— [2007]: 'Mathematical recreation versus mathematical knowledge', in Mary Leng, Alexander Paseau, and Michael Potter, eds, *Mathematical Knowledge*, pp. 109–122. Oxford: Oxford University Press.

Field, Hartry [1980]: *Science Without Numbers*. Princeton University Press.

——— [1989]: *Realism, Mathematics, and Modality*. Oxford: Basil Blackwell.

Friedman, Michael [1974]: 'Explanation and scientific understanding', *The Journal of Philosophy* **71**, 5–19.

Hafner, J., and P. Mancosu [2005]: 'The varieties of mathematical explanation', in P. Mancosu *et al.*, eds, *Visualization, Explanation and Reasoning Styles in Mathematics*, pp. 215–250. Berlin: Springer.

Hellman, Geoffrey [1989]: *Mathematics Without Numbers*. Oxford: Oxford University Press.

Hempel, Carl, and Paul Oppenheim [1948]: 'Studies in the logic of explanation', *Philosophy of Science* **15**, 135–175.

Kitcher, Philip [1981]: 'Explanatory unification', *Philosophy of Science* **48**, 507–531.

Lange, Marc [2010]: 'What are mathematical coincidences (and why does it matter)?' *Mind* **119**, 307–340.

Leng, Mary [2002]: 'What's wrong with indispensability? (Or, the case for recreational mathematics)', *Synthese* **131**, 395–417.

——— [2005]: 'Mathematical explanation', in Carlo Cellucci and Donald Gillies, eds, *Mathematical Reasoning and Heuristics*, pp. 167–189. London: King's College Publications.

——— [2010]: *Mathematics and Reality*. Oxford: Oxford University Press.

Lyon, Aidan, and Mark Colyvan [2008]: 'The explanatory power of phase spaces', *Philosophia Mathematica* (3) **16**, 227–243.

MacBride, Fraser [1999]: 'Listening to fictions: A study of Fieldian nominalism', *British Journal for the Philosophy of Science* **50**, 431–455.

Malament, David [1982]: Review of [Field, 1980], *The Journal of Philosophy* **79**, 523–534.

Mancosu, Paolo [2011]: 'Explanation in mathematics', in Edward N. Zalta, ed., *The Stanford Encyclopedia of Philosophy* (Summer 2011 Edition), http://plato.stanford.edu/archives/sum2011/entries/mathematics-explanation/ Accessed March 2014.

Melia, Joseph [1998]: 'Field's programme: Some interference', *Analysis* **58**, 63–71.

——— [2000]: 'Weaseling away the indispensability argument', *Mind* **109**, 455–479.

——— [2002]: 'Response to Colyvan', *Mind* **111**, 75–79.

——— [2010]: 'Response to Daly and Langford', *Mind* **119**, 1117–1121.

Pincock, Christopher [2004a]: 'A revealing flaw in Colyvan's indispensability argument', *Philosophy of Science* **71**, 61–79.

——— [2004b]: 'A new perspective on the problem of applying mathematics', *Philosophia Mathematica* (3) **12**, 135–161.

Putnam, Hilary [1962]: 'The analytic and the synthetic', in his *Mind, Language and Reality: Philosophical Papers, Vol. II*. Cambridge: Cambridge University Press.

——— [1971]: 'Philosophy of logic', in his *Mathematics, Matter, and Method: Philosophical Papers, Vol. I*, pp. 33–69. Cambridge: Cambridge University Press.

——— [1975]: 'What is mathematical truth?', in his *Mathematics, Matter, and Method: Philosophical Papers. Vol. I*, pp. 323–357. Cambridge: Cambridge University Press.

——— [1994]: 'Philosophy of mathematics: Why nothing works', in his *Words and Life*, pp. 499–512. Cambridge, Mass.: Harvard University Press.

Quine, W.V. [1939]: 'A logistical approach to the ontological problem', in his *The Ways of Paradox*, pp. 197–202. Cambridge, Mass.: Harvard University Press, 1976.

——— [1946]: 'Nominalism', in Dean Zimmerman, ed., *Oxford Studies in Metaphysics*. Vol. 4, pp. 3–21. Oxford: Oxford University Press, 2004.

——— [1948]: 'On what there is', in his *From a Logical Point of View*, pp. 1–19. Cambridge, Mass.: Harvard University Press, 1980.

——— [1951]: 'Two dogmas of empiricism', in his *From a Logical Point of View*, pp. 20–46. Cambridge, Mass.: Harvard University Press, 1980.

——— [1955]: 'Posits and reality', in his *The Ways of Paradox*, pp. 246–254. Cambridge, Mass.: Harvard University Press, 1976.

——— [1958]: 'Speaking of objects', in his *Ontological Relativity and Other Essays*, pp. 1–25. New York: Columbia University Press, 1969.

——— [1960]: *Word and Object*. Cambridge, Mass.: MIT Press.

——— [1978]: 'Success and the limits of mathematization', in his *Theories and Things*, pp. 148–155. Cambridge, Mass.: Harvard University Press, 1981.

——— [1986]: 'Reply to Charles Parsons', in Lewis Edwin Hahn and Paul Arthur Schilpp, eds, *The Philosophy of W.V. Quine*, pp. 396–403. La Salle, Illinois: Open Court, 1986.

Railton, Peter [1978]: 'A deductive-nomological model of probabilistic explanation', *Philosophy of Science* **45**, 206–226.

Rizza, Davide [2011]: 'Magicicada, mathematical explanation and mathematical realism', *Erkenntnis* **74**, 101–114.

Saatsi, Juha [2011]: 'The enhanced indispensability argument: Representational versus explanatory role of mathematics in science', *British Journal for the Philosophy of Science* **62**, 143–154.

Salmon, Wesley [1984]: 'Scientific explanation: Three basic conceptions', *PSA: Proceedings of the Biennial Meeting of the Philosophy of Science Association*, 1984, 293–305.

Steiner, M. [1978]:, 'Mathematical explanation', *Philosophical Studies* **34**, 135–151.

Woodward, James [2011]: 'Scientific explanation', *The Stanford Encyclopedia of Philosophy* (Winter 2011 Edition). Edward N. Zalta, ed. http://plato.stanford.edu/archives/win2011/entries/scientific-explanation/ Accessed March 2014.

Yablo, Stephen [2005]: 'The myth of the seven', in M. Kalderon, ed., *Fictionalism in Metaphysics*, pp. 88–115. Oxford: Oxford University Press.